

# The model of particle production by strong external sources

I.M. Dremin<sup>1</sup>

Lebedev Physical Institute RAS, 119991 Moscow, Russia

## Abstract

Using some knowledge of multiplicity distributions for high energy reactions, it is possible to propose a simple analytical model of particle production by strong external sources. The model describes qualitatively most peculiar properties of the distributions. The generating function of the distribution varies so drastically as it can happen at phase transitions.

PACS: 12.38Bx, 13.87.-a

## 1 Introduction

Multiparticle production is the main outcome of high energy collisions. Quantum chromodynamics (QCD) provides the general framework for its description. It is especially successful in predicting the properties of hard processes. The perturbative approach with some additional phenomenological assumptions is effective there (for reviews see [1, 2, 3]). The non-perturbative methods are waited for because the bulk of produced partons have a small fraction  $x$  of the longitudinal momentum of the incoming partners. It was proposed [4, 5, 6] to describe them as a classical color field rather than as particles. The large  $x$  partons are then considered as color sources for the classical field. Many QCD studies of this idea have been done (for recent review see [7]).

To avoid technical complications of gauge theories, it is proposed [8, 9] to consider the simpler theory of a real scalar field  $\phi$  with strong time-dependent external sources  $j$ . The Lagrangian density of the theory is

$$\mathcal{L} \equiv \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{3!} \phi^3 + j\phi. \quad (1)$$

It represents the toy model of the Color Glass Condensate formalism. One hopes that its qualitative features could also be valid for particle production in QCD. Moreover, one can use the special behavior of multiplicity distributions, gained from QCD studies and confirmed by experiment, to develop the more detailed model in the framework of such a theory. This is the main goal of the present paper.

## 2 General relations

The multiplicity distribution is the most general feature of multiparticle production processes. Any other inclusive (e.g., rapidity) distribution is obtained by averaging over different multiplicities. To be successful, any phenomenological model of these processes should, first of all, fit the experimental values for probabilities of  $n$ -particle production  $P_n$  or, equivalently, the moments of this distribution. The latter ones are easily computed with the generating function

$$G(u) = \sum_{n=0}^{\infty} P_n u^n. \quad (2)$$

---

<sup>1</sup>Email: dremin@lpi.ru

The unnormalized factorial moments are defined as

$$\mathcal{F}_q = \frac{d^q G(u)}{du^q} \Big|_{u=1} = \sum_{n=0}^{\infty} n(n-1)\dots(n-q+1)P_n, \quad (3)$$

and the unnormalized cumulant moments are

$$\mathcal{K}_q = \frac{d^q \ln G(u)}{du^q} \Big|_{u=1}. \quad (4)$$

They are connected by the iterative relations

$$\mathcal{F}_q = \sum_{m=0}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} \mathcal{K}_{q-m} \mathcal{F}_m. \quad (5)$$

The mean of the distribution of multiplicities is

$$\langle n \rangle = \sum n P_n \equiv \mathcal{F}_1 \equiv \mathcal{K}_1. \quad (6)$$

Factorial moments are always positive. Let us remind that for the Poisson distribution the normalized factorial moments  $F_q = \mathcal{F}_q / \langle n \rangle^q$  are identically equal to 1, and the normalized cumulant moments  $K_q = \mathcal{K}_q / \langle n \rangle^q$  are equal to 0 except of  $K_1 = 1$ . For NBD, both factorial and cumulant moments are positive.

It has been shown [8, 9] that in the theory defined by the Lagrangian density (1) the probability to produce  $n$  particles to all orders in the coupling  $g$  reads

$$P_n = e^{-a/g^2} \sum_{p=1}^n \frac{1}{p!} \sum_{\alpha_1+\dots+\alpha_p=n} \frac{b_{\alpha_1}\dots b_{\alpha_p}}{g^{2p}}. \quad (7)$$

Here  $a = \sum_{r=1}^{\infty} b_r$ ,  $p$  is the number of disconnected Feynman subdiagrams of the theory producing the  $n$  particles, and  $b_r/g^2$  denotes the contribution of all  $r$ -particle cuts through the connected vacuum-vacuum diagrams.

The generating function reads

$$G(u) = \exp\left[\frac{1}{g^2} \sum_{r=1}^{\infty} b_r (u^r - 1)\right]. \quad (8)$$

The average multiplicity is

$$\langle n \rangle = \frac{1}{g^2} \sum_{r=1}^{\infty} r b_r. \quad (9)$$

The cumulant moments are

$$\mathcal{K}_q = \frac{1}{g^2} \sum_{r=1}^{\infty} r(r-1)\dots(r-q+1) b_r. \quad (10)$$

Cumulant moments of  $P_n$  play the role of factorial moments of  $b_r/g$  (compare (10) and (3)). The first non-zero terms of the sum begin with  $b_q$ .

The "cut" procedure analogous to the Cutkosky rules [10] has been defined [8] for the diagrams of the theory with Lagrangian (1). The probability of having  $p$  cut subdiagrams is equal to

$$\mathcal{R}_p = \frac{1}{p!} \left(\frac{a}{g^2}\right)^p e^{-a/g^2}. \quad (11)$$

The average number of cut subgraphs is determined as

$$\langle n_{cut} \rangle = \frac{a}{g^2}. \quad (12)$$

### 3 The model

In principle, the values of  $b_r$  can be computed [8, 9] with the help of Schwinger-Keldysh formalism [11, 12] or, at leading order, in terms of a pair of solutions of the classical equations of motion. At present, this has not been done yet explicitly even though all general expressions are given in [8, 9]. Surely, some simplifications will be necessary to solve this formidable field theory problem.

In this situation, one can advocate the phenomenological approach and try to guess such behavior of  $b_r$  which would mimic the shape of the multiplicity distribution favored by QCD and experimental data. If successful, this attempt can indicate general features of  $b_r$  and help in approximate computing them from the set of Feynman diagrams.

The most astonishing regularity of behavior of multiplicity distributions predicted by QCD [13, 14, 15] and confirmed by experiment (see review [3]) is the oscillations of the cumulant moments as functions of their rank  $q$ . According to (10), this peculiar shape can be only obtained if  $b_r$  change their sign with  $r$ . To get negative values of  $b_r$ , the phases of cut diagrams in Schwinger-Keldysh approach should play important role. At least for some  $r$ ,  $b_r$  can not be interpreted as an "intrinsic"  $r$ -particle production probability as done in [8]. Correspondingly, the relation to the reggeon calculus and AGK cancellations becomes not obvious. At the same time, this oscillation should not lead to unphysical negative values of  $\langle n \rangle$  in (9).

The two-parameter  $(x, \phi)$  model with oscillating  $b_r$ , we propose to use, is

$$b_r = g^2 \frac{x^r}{r!} \cos \phi(r-1), \quad (13)$$

where

$$x = \frac{b_1}{g^2} = P_1 e^{a/g^2} > 0. \quad (14)$$

The generating function and its logarithm read

$$G(u) = \exp[e^{xu \cos \phi} \cos(xu \sin \phi - \phi) - e^{x \cos \phi} \cos(x \sin \phi - \phi)], \quad (15)$$

$$\ln G(u) = e^{xu \cos \phi} \cos(xu \sin \phi - \phi) - e^{x \cos \phi} \cos(x \sin \phi - \phi). \quad (16)$$

Note the extremely strong and rapidly varying dependence of  $G(u)$  on  $u$ . Its exponent is an exponential function itself multiplied by the oscillating factor!

The cumulant moments are given by

$$\mathcal{K}_q = x^q e^{x \cos \phi} \cos(x \sin \phi + (q-1)\phi). \quad (17)$$

They increase exponentially with  $q$  and oscillate equidistantly in the  $q$ -plane<sup>2</sup>. Their zeros  $q_m^{(0)}$  are positioned at

$$q_m^{(0)} = 1 + \frac{\pi(2m-1)}{2\phi} - \frac{x \sin \phi}{\phi}. \quad (18)$$

( $m$  is an integer number). The distance between the neighboring zeros is defined only by the parameter  $\phi$

$$\Delta q^{(0)} = \frac{\pi}{\phi}. \quad (19)$$

---

<sup>2</sup>For NBD, they increase even faster (as  $q!$ ) and do not oscillate being always positive.

The average multiplicity is

$$\langle n \rangle \equiv \mathcal{K}_1 = x e^{x \cos \phi} \cos(x \sin \phi). \quad (20)$$

The necessary requirement of positive  $\langle n \rangle$  asks for  $x \sin \phi$  to be positioned in the hemispheres where  $\cos(x \sin \phi)$  is positive. Moreover, from the requirement  $q_1^{(0)} > 1$  one obtains for  $\phi > 0$

$$0 < x \sin \phi < \pi/2. \quad (21)$$

The probability of having  $p$  cut subdiagrams (11) is determined by

$$\frac{a}{g^2} = e^{x \cos \phi} \cos(x \sin \phi - \phi) - \cos \phi. \quad (22)$$

Since both  $\langle n \rangle$  and  $q_m^{(0)}$  depend on energy, the parameters  $x$  and  $\phi$  are also functions of energy. However, the product  $x \sin \phi$  must be constant because any its dependence would induce quite strange "quasi-oscillating" component in energy behavior of the mean multiplicity. Both  $\langle n \rangle$  and  $q_m^{(0)}$  should increase with energy tending asymptotically to infinity. For  $q_m^{(0)}$ , this property follows from QCD asymptotics of  $K_q$  where the leading order results survive and all cumulant moments become positive. Then asymptotically  $\phi$  tends to 0 and  $x$  tends to infinity so that the product  $x\phi$  stays (or tends to) constant. For  $\langle n \rangle$  and  $q_1^{(0)}$  one obtains

$$\langle n \rangle = x e^x \cos(x\phi), \quad (23)$$

$$q_1^{(0)} = 1 + \frac{\pi - 2x\phi}{\phi}. \quad (24)$$

If  $x\phi \rightarrow 0$ , then the asymptotics of  $\langle n \rangle$  and  $q_1^{(0)}$  are defined by the parameters  $x$  and  $\phi$  correspondingly and, therefore, completely independent.

The QCD-like results for the energy dependence of  $\langle n \rangle$  can be obtained with  $x \propto \sqrt{\ln s}$  at  $s \rightarrow \infty$  (up to NLO corrections). For  $x \propto \ln s$ , one gets the power increase of the mean multiplicity with energy typical for hydrodynamics [16] and fixed coupling QCD [17, 18].

Inserting ansatz (13) in Eq. (7) one gets the general form of the multiplicity distribution in terms of  $x$  and  $\phi$

$$P_n = \frac{x^n}{n!} f_n(\phi), \quad (25)$$

where

$$f_n(\phi) = \sum_{k=0}^{n-1} c_{n,k} \cos^k \phi. \quad (26)$$

Unfortunately, no analytical expression for the coefficients  $c_{n,k}$  is obtained. For lowest multiplicities, they are  $c_{2,0} = 1, c_{2,1} = 1; c_{3,0} = 0, c_{3,1} = 3, c_{3,2} = 2; c_{4,0} = 2, c_{4,1} = 1, c_{4,2} = 11, c_{4,3} = 4$ .

The model is completely fixed if its parameters  $x$  and  $\phi$  are replaced by the average multiplicity and the position of the first zero of cumulant moments. By the order of magnitude, these values are similar in different reactions at high energies [19, 20]. To be more definitive, let us use our experience in  $e^+e^-$ -collisions at LEP energies [3]. There, the first zero of cumulant moments  $q_1^{(0)}$  is located near the value  $q = 4$ . The average total multiplicity is about 40. These values can be approximately obtained with the model parameters

$$x = 3, \quad \phi = \pi/12. \quad (27)$$

The only inconsistency with experiment comes from the distance between zeros of cumulant moments which equals 12 that is much larger than the experimentally observed ones. However,

it would be very naive to expect even this quite exotic quantity to be perfectly reproduced by such a simple phenomenological model. Further refinements of the expression (13) are possible to get better agreement (e.g., the nonlinear dependence of the cosine argument on  $r$ ).

Another problem appears for hadronic reactions if the same parameters are used. Particle production is often interpreted in terms of reggeon exchanges. In [8] it is proposed to identify cut disconnected vacuum-vacuum diagrams as cut reggeons. Then the average number of these diagrams (12) for the above values of  $x$  and  $\phi$  is very large  $\langle n_{cut} \rangle \approx 14$  compared to the typical values in widely used reggeon models, and the average number of particles in a single cut subgraph is quite small  $\langle n \rangle_1 \approx 3$ . Therefore, the simple identification of cut subdiagrams with cut reggeons looks rather improbable.

One of the most important features of the model is the violent behavior of the generating function in the complex  $u$ -plane. Its extrema are positioned at

$$u_k = \frac{\pi(2k-1)}{2x \sin \phi} \quad (28)$$

( $k$  is an integer number) with maxima and minima replacing one another at each subsequent  $k$  and separated by the distance

$$\Delta u = \frac{\pi}{x \sin \phi}. \quad (29)$$

It is especially interesting that the first extremum  $u_1$  is positioned at  $u_1 = 2$  for the above chosen values of the parameters  $x$  and  $\phi$ . The generating function reaches extremely high value at this point

$$G(u_1) \equiv G(2) \approx \exp\left[\frac{\pi}{12}e^6\right] \gg 1. \quad (30)$$

At the next minimum  $u_2 = 6$  the generating function is very small

$$G(u_2) \equiv G(6) \approx \exp\left[-\frac{\pi}{12}e^{18}\right] \ll 1. \quad (31)$$

Maxima and minima are separated by  $\Delta u = 4$ .

In some way the generating function reminds the grand canonical function of the statistical mechanics with  $z = u - 1$  interpreted as fugacity. It was shown [21, 22] that the singularity of the grand canonical function at  $u = 2$  corresponds to the phase transition in the condensed matter. The obtained large value of  $G(2)$  indicates that the model probably describes the processes with drastically varying characteristics. The point  $u = 2$  is the accumulating point of zeros of the cut generating functions

$$G_c(u) = \sum_{n=1}^{n_{max}} P_n u^n \quad (32)$$

for  $n_{max} \rightarrow \infty$ . These zeros are located [23, 24] near the circle  $|u - 1| = 1$  in the complex  $u$ -plane. Actually, only the cut generating functions are available from experiment because of finite  $n_{max}$  measurable at finite energies. It is the polynomial of  $u$  normalized at the center of the circle  $|u - 1| = 1$  by the condition  $G_c(1) = 1$  with zero at  $u = 0$  and other  $n_{max} - 1$  complex conjugated zeros near this circle. For example, for  $G_c(n_{max} = 3)$  these two zeros are at

$$u_{2,3} = -\frac{P_2}{2P_3} \pm i \frac{\sqrt{4P_3P_1 - P_2^2}}{2P_3} \quad (33)$$

with

$$|u_{2,3}|^2 = \frac{P_1}{P_3} \ (\ll 1), \quad \text{Re}u_{2,3} = -\frac{P_2}{2P_3} < 0. \quad (34)$$

At larger  $n_{max}$ ,  $\text{Re}U_{n_{max}}$  increases and tends to 2 with  $\text{Im}u_{n_{max}}$  coming closer to the circle  $|u - 1| = 1$ . It is remarkable that accumulation of zeros is related to the very high value of  $G(2)$  which reminds the singular behavior at phase transitions.

## 4 Conclusions

The simple analytical model of particle production by strong external sources is proposed. All the moments of multiplicity distribution and its generating function are calculable analytically. It rarely happens that one gets them altogether. This is the privilege of simplest distributions of the probability theory but not of QCD.

The model describes qualitatively the multiplicity distribution of produced particles fitting its mean value and higher order moments to experimental data and QCD findings. In particular, the oscillations of cumulant moments, first predicted by QCD, are reproduced. The energy behavior of these characteristics is related to the energy dependence of the model parameters. Asymptotic relations help reveal the tendencies in this dependence.

The generating function of the distribution sharply depends on the auxiliary variable  $u$ . This behavior is reminiscent of the grand canonical function in statistical mechanics near the phase transitions. The exponential function whose exponent is an exponential function by itself with an oscillating factor in front of it is an unique guest in physics studies. It has no analogues in the well known distributions of the probability theory either.

Even though the results rely strongly on the expression (8) obtained for  $\phi^3$ -theory with strong external sources, they can be of more general validity. The ansatz (13) combined with (8) reproduces very tiny features of multiplicity distributions learned from QCD and experimental studies. It can help get some insight for developing approximate methods of computing these probabilities in the field theory approach.

## 5 Acknowledgments

This work has been supported in part by the RFBR grants 04-02-16445-a, 04-02-16333, 06-02-17051.

## References

- [1] I.M. Dremin, Uspekhi Fiz. Nauk 164 (1994) 785; Physics-Uspekhi 37 (1994) 715.
- [2] V.A. Khoze, W. Ochs, Int. J. Mod. Phys. A12 (1997) 2949.
- [3] I.M. Dremin, J.W. Gary, Phys. Rep. 349 (2001) 301.
- [4] L.D. McLerran, R. Venugopalan, Phys. Rev. D49 (1994) 2233.
- [5] L.D. McLerran, R. Venugopalan, Phys. Rev. D49 (1994) 3352.

- [6] L.D. McLerran, R. Venugopalan, Phys. Rev. D50 (1994) 2225.
- [7] A.V. Leonidov, Uspekhi Fiz. Nauk 175 (2005) 345; Physics-Uspekhi 48 (2005) 323.
- [8] F. Gelis, R. Venugopalan, hep-ph/0601209.
- [9] F. Gelis, R. Venugopalan, hep-ph/0605246.
- [10] R.E. Cutkosky, J. Math. Phys. 1 (1960) 429.
- [11] J. Schwinger, J. Math. Phys. 2 (1961) 407.
- [12] L.V. Keldysh, Sov. Phys. JETP 20 (1964) 1018.
- [13] I.M. Dremin, Phys. Lett. B313 (1993) 209.
- [14] I.M. Dremin, V.A. Nechitailo, JETP Lett. 58 (1993) 945.
- [15] I.M. Dremin, V.A. Nechitailo, Mod. Phys. Lett. A9 (1994) 1471.
- [16] L.D. Landau, Izv. Akad Nauk SSSR, Ser. Fiz., 17 (1953) 51.
- [17] I.M. Dremin, R.C. Hwa, Phys. Rev. D49 (1994) 5805.
- [18] I.M. Dremin, R.C. Hwa, Phys. Lett. B324 (1994) 477.
- [19] I.M. Dremin, V.A. Nechitailo, M. Biyajima, N. Suzuki, Phys. Lett. B403 (1997) 149.
- [20] E.K.G. Sarkisyan, A.S. Sakharov, AIP Conf. Proc. 828 (2006) 35; hep-ph/0410324.
- [21] T.D. Lee, C.N. Yang, Phys. Rev. D87 (1952) 404.
- [22] C.N. Yang, T.D. Lee, Phys. Rev. D87 (1952) 410.
- [23] I.M. Dremin, Pisma ZhETF 60 (1994) 757; JETP Lett. 60 (1994) 764.
- [24] E.A. DeWolf, in Proc. XXIV Int. Symposium on Multiparticle Dynamics (Vietri sul Mare, Italy, 1994), Eds. A Giovannini, S. Lupia, A. Ugoccioni, WSPC, Singapore, 1995, p.15.